<https://www.rd.com/article/logic-puzzles/>

**6. ALLERGY SEASON:**

Five friends (Allegra, Ben, Clara, Flora, and Zach) are each allergic to something different: pollen, shellfish, bee stings, cats, or nuts.

* Allegra has a food allergy
* Ben can play with his kitten for hours without issue (or medicine).
* Clara’s allergy is not related to animals.
* Flora has seasonal allergies.

Can you figure out who is allergic to what?

**SMT ENCODING STRATEGY**

We will model this problem using LIA (Linear Integer Arithmetic). Each Allergy is assigned to a number 1-5: pollen=1, shellfish=2, beestings=3, cats=4, and nuts=5. Each friend will be modeled as a variable with relevant assignments and constraints.

**SMT ENCODING**

; Variable declarations

(declare-fun a () Int)

(declare-fun b () Int)

(declare-fun c () Int)

(declare-fun f () Int)

(declare-fun z () Int)

; Constraints

(assert (> a 0)) ; Bounding all outputs 1-5

(assert (< a 6))

(assert (> b 0))

(assert (< b 6))

(assert (> c 0))

(assert (< c 6))

(assert (> f 0))

(assert (< f 6))

(assert (> z 0))

(assert (< z 6))

;1=pollen, 2=shellfish, 3=beestings, 4=cats, 5=nuts

(assert (not (= a b))) ; ensuring mutual exclusivity

(assert (not (= a c)))

(assert (not (= a f)))

(assert (not (= a z)))

(assert (not (= b c)))

(assert (not (= b f)))

(assert (not (= b z)))

(assert (not (= c f)))

(assert (not (= c z)))

(assert (not (= f z)))

(assert (or (= a 2) (= a 5)));Allegra has a food allergy

(assert (not (= b 4))); Ben is not allergic to cats

; Clara's allergy is not related to animals

(assert (not (or (= c 4) (= c 3))))

(assert (not (= c 2) ))

(assert (= f 1)) ;Flora has seasonal allergies

; Solve

(check-sat)

(get-model)

**SMT OUTPUT**

sat

(model

(define-fun z () Int

4)

(define-fun b () Int

3)

(define-fun a () Int

2)

(define-fun c () Int

5)

(define-fun f () Int

1)

)

**INTERPRETING THE OUTPUT**

To interpret the output we just need to take the variables and numbers and assign them to their relevant names and allergies. So, Zach is allergic to cats, Ben is allergic to bee stings, Allegra is allergic to shellfish, Clara is allergic to nuts, Flora is allergic to pollen.

**PROVIDED ANSWER:** Allegra is allergic to shellfish, Ben to bee stings, Clara to nuts, Flora to pollen, and Zach to cats.

**Further Notes**

The code written is very repetitive, with ~10 lines dedicated to making sure that all the values are unique, and another 10 lines dedicated to bounding all the variables. This algorithm scales poorly due to the uniqueness guarantee, each additional variable added means an additional line of code for every variable already added. I also do not think bees count as animals, but the only way for the algorithm to work is if we set Clara as not allergic to cats, shellfish, and bee stings.